Published by Institute of Physics Publishing for SISSA

Received: *January 6, 2008*
Accepted: *February 5, 2008*
Published: *February 14, 2008*

First-order attractor flow equations for supersymmetric black rings in $N = 2$, $D = 5$ supergravity

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Abstract: In this paper we investigate the attractor mechanism in the five dimensional low energy supergravity theory corresponding to M-theory compactified on a Calabi-Yau threefold CY_3. Using very special geometry, we derive the general first-order attractor flow equations for BPS and non-BPS solutions in five-dimensional Gibbons-Hawking spaces. Especially, considering the supersymmetric solution, we obtain the first-order flow equations for supersymmetric (multi)black rings. We also solve the flow equations and discuss some properties of the solutions of flow equations.

Keywords: Field Theories in Higher Dimensions, Black Holes in String Theory, M-Theory, Supergravity Models.

Contents

1. Introduction

The attractor mechanism in extremal black holes has been an interesting subject over the past few years, which states that the values of the moduli scalar fields at the horizon of the extremal black holes are independent of the asymptotic values for the moduli fields and are entirely determined by the quantized charges of the black holes. It was initiated in the context of $N = 2$ supergravity theories in four dimensions [1], then extended to other supergravity theories and superstring theories, such as supersymmetric black holes with higher derivative corrections , multi-center black holes and spherically symmetric or rotating black holes in higher dimensions [2]. More recently, more attention has been paid to non-supersymmetric cases [4, 3].

In this mechanism, there exists a set of first-order differential equations, known as "attractor flow equations", which describe the evolution of the spacetime metric and the moduli fields in the background of extremal black holes. In the literature, there are two methods to obtain these equations: one can follow the method in [1] , imposing the preservation of supersymmetry, the gravitino and gaugino variations vanish and lead to a set of first-order flow equations about the metric and moduli fields . Another alternative method is advised by Ferrara *et al.* in [3], which is that one can consider the attractor flow behavior as a result of an extremization of the effective Lagrangian, rather than a supersymmetric flow.

As is well known, the horizon topology of black holes in five dimensional spacetime is not unique. The discovery of a new BH phase made by Emparan and Reall [5]: an asymptotically flat, rotating black hole solution with horizon topology $S^1 \times S^2$ and carrying angular momentum along the S^1, is called as black ring. Several important developments are listed in [6 – 8]. For reviews, see [9].

It is interesting to understand the attractor mechanism in the context of black ring. This mechanism has been addressed by Kraus and Larsen in [10]. In particular, by examining the the BPS equations for black rings , they found the flow equations for supersymmetric extremal black rings, an equation relating the flow of the moduli to changes in the gauge field. The attractor mechanism for the black rings determines the scalar values at the near-horizon region via the magnetic dipole charges only. But, the flow equation in [10] is a second-order differential equation. It is obvious to mention that do the first-order differential flow equations analogous to the equations in [1, 4] can exit and, if so, how?

This is the aim of this article to further study the attractor mechanism in five dimensional black rings. Recently, based on very special geometry, Cardoso *et al*. in [11] proposed an effective method for deriving first-order flow equations for rotating electrically charger extremal black holes in five dimensions. Inspired by [11], we construct the general first-order attractor flow equations for BPS and non-BPS solutions in five dimensional Gibbons-Hawking spaces by making use of the stationarity of actions. Especially, considering the configuration of supersymmetric solution, we obtain the first-order attractor equations for supersymmetric (multi)black rings, which take the form analogous to that of a gradient flow of black holes in five dimensions [11, 12]. Because we do not analyze the attractor flow equations following the method in [10], we are still uncertain whether the second-order equation in [10] can be reduced to a set of first-order flow equations. However, when we consider the condition of supersymmetric solutions, the first-order flow equations which we obtain in five dimensional Gibbons-Hawking spaces can reproduction a second-order equation the flow equation derived in [9]. By integrating the first-order flow equations, we also find the relation between the flow equations and electronic central charge Z_e corresponding to the graviphoton.

This paper is organized as follows. In the next section, we briefly review the supersymmetric solutions of $N = 2$ supergravity and specialized to the case of (multi)black rings in the Gibbons-Hawking spaces. Using very special geometry and the condition of stationary of action, the generalized first-order attractor flow equations are carried out in section 3. In section 4 we present some properties of flow equations for supersymmetric black rings and the example of the limit of black rings is given. The last section is devoted to discussions.

2. A brief review of supersymmetric solutions of $N = 2$ supergravity

In this section we first present a brief review of the five dimensional low energy supergravity theory corresponding to M-theory compactified on a Calabi-Yau threefold CY_3. This model is usually studied in the context of real or very special geometry. Relevant references can be found in [13 – 16]. We also introduce the general supersymmetric solutions in Gibbons-Hawking base space. Further details, see [7].

2.1 $N = 2$ supergravity

The bosonic part of $N = 2$ $D = 5$ ungauged supergravity coupled to $n - 1$ abelian vector multiplets with scalars $\phi^i, i = 1, \ldots, n-1$, is

$$S = \frac{1}{16\pi G_5} \int \Big(R * 1 - G_{IJ} F^I \wedge *F^J - G_{IJ} dX^I \wedge *dX^J \\ - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K \Big) , \tag{2.1}$$

where the scalars $X^I = X^I(\phi^i)$, $I, J, K = 1, \ldots, n$, obey the constraint: $\frac{1}{6} C_{IJK} X^I X^J X^K = 1$ and the constants C_{IJK} are symmetric on IJK. It is useful to define

$$X_I = \frac{1}{6} C_{IJK} X^J X^K , \tag{2.2}$$

$$G_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K . \tag{2.3}$$

From the definitions, it follows that

$$X^I X_I = 1 , \quad X_A = \frac{2}{3} G_{AB} X^B , \quad X^A = \frac{3}{2} G^{AB} X_B , \tag{2.4}$$

and so

$$X^I \partial_i X_I = \partial_i X^I X_I = 0 . \tag{2.5}$$

By the definition (2.3) one can find

$$\partial_i X_I = -\frac{2}{3} G_{IJ} \partial_i X^J , \quad \partial_i X^I = -\frac{3}{2} G^{IJ} \partial_i X_J . \tag{2.6}$$

The metric g_{ij} on the scalar manifold is

$$g_{ij} = G_{AB} \partial_i X^A \partial_j X^B , \tag{2.7}$$

where $\partial_i X^I = \frac{\partial X^I}{\partial \phi^i}$. Combing these relations with the index structure we find that

$$g^{ij} \partial_i X^A \partial_j X^B = a(G^{AB} - b X^A X^B) , \tag{2.8}$$

with constant coefficients $a = 1$ and $b = \frac{2}{3}$.

We are interested in solutions preserving some supersymmetry. Following the reference [17, 18], supersymmetric solutions of five-dimensional supergravity imply the existence of a non-spacelike Killing vector field, and assuming that in a region the Killing vector field $V = \partial/\partial t$ is time-like , the $D = 5$ metric is given by

$$ds_5^2 = -f^2 (dt + \omega)^2 + f^{-1} ds^2_{\mathcal{M}} , \tag{2.9}$$

where $\mathcal{M}$ is a four-dimensional hyper-Kähler manifold, and f and ω are a scalar and a 1-form on $\mathcal{M}$. The field strengths F^I can be written

$$F^I = d\left[(f X^I (dt + \omega) \right] + \Theta^I , \tag{2.10}$$

where Θ^I are closed 2-forms on the $\mathcal{M}$. The scalar function X_I and f, one-forms ω and Θ^I on $\mathcal{M}$ are given by

$$(\Theta^I)^- = 0 , \quad \triangle_{\mathcal{M}} (f^{-1} X_I) = \frac{1}{6} C_{IJK} \Theta^J \cdot \Theta^K , \quad (d\omega)^+ = -\frac{3}{2} f^{-1} X_I \Theta^I , \tag{2.11}$$

where $\triangle_{\mathcal{M}}$, the Laplacian, and the superscripts $\pm$, self-dual and antiself-dual , are defined with respect to the base $\mathcal{M}$, and for 2-forms α and β on $\mathcal{M}$ we define $\alpha \cdot \beta = \alpha^{mn} \beta_{mn}$, with indices raised by the matrix h^{mn} on $\mathcal{M}$. This equations are named as BPS equations.

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2.2 Gibbons-Hawking base spaces and black ring solutions

Now let us concentrate on the so-called Gibbons-Hawking base spaces. In this paper, as the base space $\mathcal{M}$, we consider the Gibbons-Hawking metric, which can then be written as

$$ds^2_{\mathcal{M}} = H^{-1}(dx^5 + \chi)^2 + H\delta_{ij}dx^j dx^j\,, \tag{2.12}$$

where H is harmonic on the Euclid space $\mathbb{E}^3$, $\chi = \chi_i dx^i$, $i, j = 1, 2, 3$, and H, χ are independent of x^5 and can be solved explicitly, χ is determined by $\nabla \times \boldsymbol{\chi} = \nabla H$. In this section ∇ will be the gradient and ∇^2 will be the Laplacian on $\mathbb{E}^3$.

We introduce one-forms η^I, $\Theta^I = d\eta^I$. It is convenient to set

$$\begin{aligned} \omega &= \omega_5\left(dx^5 + \chi\right) + \hat{\omega}_4\,, & (2.13)\\ \eta^I &= \eta^I_5\left(dx^5 + \chi\right) + \hat{\eta}^I_4\,, & (2.14)\end{aligned}$$

where $\hat{\omega}_4 = \omega_{4i}dx^i$, $\hat{\eta}^I_4 = \eta^I_{4i}dx^i$. We can solve the BPS equation and obtain [7]:

$$\begin{aligned} \nabla \times \hat{\eta}^I_4 &= -\nabla\left(H\eta^I_5\right)\,, & (2.15)\\ \nabla \times \hat{\omega}_4 &= H\nabla\omega_5 - \omega_5\nabla H + 3\,H\big(f^{-1}X_I\big)\nabla\eta^I_5\,, & (2.16)\\ f^{-1}X_I &= \frac{1}{24}H^{-1}C_{IPQ}K^PK^Q + L_I, & (2.17)\end{aligned}$$

where $\eta^I_5 = \frac{1}{2}H^{-1}K^I$, and L_I, K^I are harmonic functions on $\mathbb{E}^3$. Using the integrability condition of equation (2.16), we find the constraint

$$\nabla^2\omega_5 = \nabla^2\left(-\frac{1}{48}H^{-2}C_{IPQ}K^IK^PK^Q - \frac{3}{4}H^{-1}L_IK^I\right)\,. \tag{2.18}$$

The solution of this equation is read as

$$\omega_5 = -\frac{1}{48}H^{-2}C_{IPQ}K^IK^PK^Q - \frac{3}{4}H^{-1}L_IK^I + B\,, \tag{2.19}$$

where B is another harmonic function on $\mathbb{E}^3$. The general solution with Gibbons-Hawking base is specified by $2n + 2$ harmonic functions H, K^I, L_I and B on $\mathbb{E}^3$. It is well known that H determines the Gibbons-Hawking base, such as three examples of Gibbons-Hawking metrics: flat space ($H = 1$ or $H = 1/|\mathbf{x}|$), Taub-NUT space ($H = 1 + 2M/|\mathbf{x}|$) and the Eguchi-Hanson space ($H = 2M/|\mathbf{x}| + 2M/|\mathbf{x} - \mathbf{x_0}|$) [17]. It is convenient to take the base space $\mathcal{M}$ to be flat space $\mathbb{E}^4$ with metric

$$ds^2_{\mathbb{E}^4} = H^{-1}(d\psi + \chi)^2 + H(dr^2 + r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right])\,, \tag{2.20}$$

where $H = 1/|\mathbf{x}| \equiv 1/r$ and $\chi = \cos\theta d\phi$, which satisfies $\nabla \times \boldsymbol{\chi} = \nabla H$. The range of the angular coordinates are $0 < \theta < \pi$, $0 < \phi < 2\pi$ and $0 < \psi < 4\pi$.

We are interested in the solutions of supersymmetric black ring (multi-black rings). In [7] the multi-black rings solutions is given by

$$
\begin{aligned}
K^I &= \sum_{i=1}^{M} q^I{}_i h_i \,, \\
L_I &= \lambda_I + \frac{1}{24}\sum_{i=1}^{M}\left(Q_{Ii} - C_{IJK} q^J{}_i q^K{}_i\right) h_i \,, \\
B &= \frac{3}{4}\sum_{i=1}^{M} \lambda_I q^I{}_i - \frac{3}{4}\sum_{i=1}^{M} \lambda_I q^I{}_i |\mathbf{x}_i| h_i \,,
\end{aligned}
\tag{2.21}
$$

where h_i are harmonic functions in $\mathbb{E}^3$ centred at $\mathbf{x}_i$, $h_i = 1/|\mathbf{x} - \mathbf{x}_i|$, and Q_{Ii}, q^I_i and λ_I are constants.

3. First-order flow equations in Gibbons-Hawking spaces

In this section, we derive the general first-order attractor flow equations for BPS and non-BPS solutions in five dimensional Gibbons-Hawking spaces with the metric (2.20), following the method of [10]. To simplify the calculation , we assume that all black rings are sitting along the negative z-axis of the three-dimensional space, ie, $\mathbf{x}_i$ located along the z-axis , then, K^I, L_I, B in (2.21)only depend on r, θ.

Take the five-dimensional configuration as follows:

$$
\begin{aligned}
ds_5^2 &= G_{MN}\, dx^M\, dx^N = -f^2(r,\theta)\,(dt+w)^2 + f^{-1}(r,\theta)\, ds^2_{\mathbb{E}^4} \,, \\
A^I &= \chi^I(r,\theta)\,(dt+w) + \eta^I \,, \\
w &= w_5(r,\theta)\,(d\psi + \cos\theta\, d\phi) + w_4(r,\theta)\, d\phi \,, \\
\eta^I &= \eta^I_5(r,\theta)\,(d\psi + \cos\theta\, d\phi) + \eta^I_4(r,\theta)\, d\phi \,,
\end{aligned}
\tag{3.1}
$$

where $ds^2_{\mathbb{E}^4}$ is the metric (2.20). Substituting (3.1) into the action (2.1), we find that the bosonic part of the five-dimensional $N = 2$ supergravity action can be expressed as

$$
\frac{8\pi G_5}{V}\, S = S_1 + S_2 + S_3 + S_4 \,, \tag{3.2}
$$

with $V = 1/2$. Our goal is to find the first-order attractor flow equations from this actions. We can proceed in the following steps. Step one: The action S is express in terms of squares of first-order terms and total derivative terms. Then, step two: Stationarity of action S is imposed which implies all the first-order terms vanish. After a tedious calculation, we obtain (we refer to appendix for some of the details)

$$
\begin{aligned}
S_1 = \frac{1}{2}\int & dt\, dr\, d\theta\, d\phi\, d\psi \\
&\Big[\sin\theta\Big(-3r^2 f^{-2}\partial_r^2 f - 2r^2 g_{ij}\partial_r\phi^i\partial_r\phi^j + 2r^2 f^{-2} G_{AB}\partial_r\chi^A\,\partial_r\chi^B \\
&\qquad +2\partial_r\left(r^2 f^{-1}\partial_r f\right)\Big) + \Big(-3\sin\theta f^{-2}\partial_\theta^2 f - 2\sin\theta g_{ij}\partial_\theta\phi^i\partial_\theta\phi^j \\
&\qquad\qquad +2\sin\theta f^{-2} G_{AB}\partial_\theta\chi^A\,\partial_\theta\chi^B + 2\partial_\theta\left(\sin\theta f^{-1}\partial_\theta f\right)\Big)\Big] \,,
\end{aligned}
\tag{3.3}
$$

This action is a function of the spacetime metric f and the moduli ϕ, including their derivatives. It can lead to the Einstein equation and scalar equations of motion.

$$
\begin{aligned}
S_2 = \frac{1}{2}\int & dt\,dr\,d\theta\,d\phi\,d\psi\,\sin\theta \\
& \Big[-2\frac{f}{r}G_{AB}(\eta_5^A\eta_5^B + \partial_\theta\eta_5^A\partial_\theta\eta_5^B - 2\csc\theta\,\eta_5^A\partial_\theta\eta_4^B \\
&\qquad + \csc^2\theta\,\partial_\theta\eta_4^A\partial_\theta\eta_4^B + r^2\,\partial_r\eta_5^A\partial_r\eta_5^B + r^2\csc^2\theta\,\partial_r\eta_4^A\partial_r\eta_4^B) \\
&\quad +4\frac{sf}{\sin\theta}G_{AB}\Big(\partial_r(\eta_5^B\cos\theta + \eta_4^B)\partial_\theta\eta_5^A - \partial_\theta(\eta_5^B\cos\theta + \eta_4^B)\partial_r\eta_5^A\Big)\Big]\,.
\end{aligned} \tag{3.4}
$$

This action is a function of the gauge potential η^I, and its derivatives.

$$
\begin{aligned}
S_3 = \int & dt\,dr\,d\theta\,d\phi\,d\psi \\
& \Big[\frac{f\sin\theta}{2r}\Big((w_5^2 + \partial_\theta^2 w_5 - 2\csc\theta\,w_5\partial_\theta w_4 + \csc^2\theta\,\partial_\theta w_4^2 + r^2\,\partial_r w_5^2 \\
&\quad +r^2\csc^2\theta\,\partial_r w_4^2)(f^2 - 2G_{AB}\chi^A\chi^B) - 4G_{AB}\chi^A(\partial_\theta\eta_5^B\,\partial_\theta w_5 \\
&\quad +\csc^2\theta\,\partial_\theta\eta_4^B\,\partial_\theta w_4 - \csc\theta w_5\,\partial_\theta\eta_4^B + \eta_5^B w_5 + r^2\,\partial_r\eta_5^B\partial_r w_5 \\
&\quad +r^2\csc^2\theta\,\partial_r\eta_4^B\partial_r w_4)\Big) + \frac{1}{3V}C_{ABC}\chi^A\chi^B\chi^C\Big((w_5\sin\theta - \partial_\theta w_4)\partial_r w_5 \\
&\quad +\partial_r w_4\partial_\theta w_5\Big) - \frac{1}{2V}C_{ABC}\chi^A\chi^B\Big(\partial_r w_5\partial_\theta\eta_4^C - \eta_5^C\sin\theta\partial_r w_5 \\
&\quad -\partial_\theta w_5\partial_r\eta_4^C + \partial_\theta w_4\partial_r\eta_5^C - \partial_r w_4\partial_\theta\eta_5^C - w_5\sin\theta\partial_r\eta_5^C\Big) \\
&\quad -9\frac{X_AX_B}{V^2}sf\Big(\partial_r(\eta_5^B\cos\theta + \eta_4^B)\partial_\theta\eta_5^A - \partial_\theta(\eta_5^B\cos\theta + \eta_4^B)\partial_r\eta_5^A\Big)\Big]
\end{aligned} \tag{3.5}
$$

and

$$
\begin{aligned}
S_4 = \int & dt\,dr\,d\theta\,d\phi\,d\psi \\
& \frac{1}{3V}C_{ABC}\Big[\frac{1}{2}\partial_r\Big(\chi^A\chi^B\eta_5^C\partial_\theta(w_5\cos\theta + w_4) - \chi^A\chi^B(\eta_5^C\cos\theta + \eta_4^C)\partial_\theta w_5 \\
&\qquad +2\chi^A(\eta_5^B\partial_\theta\eta_4^C - \eta_5^B\eta_5^C\sin\theta - \partial_\theta\eta_5^B\eta_4^C)\Big) \\
&\quad -\frac{1}{2}\partial_\theta\Big(\chi^A\chi^B\eta_5^C\partial_r(w_5\cos\theta + w_4) - \chi^A\chi^B(\eta_5^C\cos\theta + \eta_4^C)\partial_r w_5 \\
&\qquad +2\chi^A(\eta_5^B\partial_r\eta_4^C - \partial_r\eta_5^B\eta_4^C)\Big)\Big]\,.
\end{aligned} \tag{3.6}
$$

S_3 contains terms that are proportional to η^I and w and derivatives thereof. S_4 is composed of the total derivative terms, so ,we call S_4 as the boundary term. S_2 and S_3 determine the evolution of the gauge potential A^I and w . We can also find that all four parts of the action can split into two parts: one about coordinate r and another about θ.

The terms in S_1 can be written as:

$$
S_1 = S_1^{(a)} + S_1^{(b)}\,, \tag{3.7}
$$

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$$
\begin{aligned}
S_1^{(a)} = \frac{1}{2}\int dt\, dr\, d\theta\, d\phi\, d\psi\, \sin\theta \big[&-3r^2 f^{-2}(\partial_r f)^2 - 2r^2 g_{ij}\partial_r\phi^i\partial_r\phi^j \\
&+2r^{-2}f^{-2}G_{AB}\left(r^2\partial_r\chi^A + f^2 G^{AC}V_C\right)\left(r^2\partial_r\chi^B + f^2 G^{BD}V_D\right) \\
&-2r^{-2}f^2\, V_A G^{AB}V_B + 2\partial_r\left(r^2 f^{-1}\partial_r f - 2V_A\,\chi^A\right) + 4\chi^A\partial_r V_A\big]\ , \\
S_1^{(b)} = \frac{1}{2}\int dt\, dr\, d\theta\, d\phi\, d\psi\, \big[&-3\sin\theta f^{-2}\partial_\theta^2 f - 2\sin\theta g_{ij}\partial_\theta\phi^i\partial_\theta\phi^j \\
&+2\csc\theta f^{-2}G_{AB}\left(\sin\theta\partial_\theta\chi^A + f^2G^{AC}U_C\right)\left(\sin\theta\partial_\theta\chi^B + f^2G^{BD}U_D\right) \\
&-2\csc\theta f^2 U_A G^{AB}U_B + 2\partial_\theta\left(\sin\theta f^{-1}\partial_\theta f - 2U_A\chi^A\right) + 4\chi^A\partial_\theta U_A\big]\ ,
\end{aligned} \tag{3.8}
$$

where $V = V(r,\theta)$ and $U = U(r,\theta)$ are scalar functions. The term proportional to $V_A G^{AB} V_B$, with the definition of (2.8), can be written as

$$
V_A G^{AB} V_B = \frac{2}{3} V_A\, X^A X^B V_B + g^{ij}\, V_A \partial_i X^A\, \partial_j X^B V_B\ , \tag{3.9}
$$

we can also obtain the similar result about the term $U_A G^{AB} U_B$. Then, using these relations, we obtain

$$
\begin{aligned}
S_1^a = \frac{1}{2}\int dt\, dr\, d\theta\, d\phi\, d\psi\, \sin\theta \Bigg[&-3\tau^2 f^2\left(\partial_\tau f^{-1} - \frac{2}{3}V_A X^A\right)^2 \\
&-2\tau^2 g_{ij}\left(\partial_\tau\phi^i + f g^{il}V_A\partial_l X^A\right)\left(\partial_\tau\phi^j + f g^{jk}V_B\partial_k X^B\right) \\
&+2\tau^2 f^{-2}G_{AB}\left(\partial_\tau\chi^A - f^2G^{AC}V_C\right)\left(\partial_\tau\chi^B - f^2G^{BD}V_D\right) \\
&+2\partial_r\left(r^2 f^{-1}f' - 2V_A\,\chi^A - 2f\,V_A X^A\right) + 4\chi^A\partial_r V_A + 4fX^A\partial_r V_A\Bigg],
\end{aligned} \tag{3.10}
$$

where $\tau = \frac{1}{r}$.

$$
\begin{aligned}
S_1^b = \frac{1}{2}\int dt\, dr\, d\theta\, d\phi\, d\psi\, \Bigg[&-3\csc\theta\, f^2\left(\sin\theta\, f^{-2}\partial_\theta f - \frac{2}{3}U_A X^A\right)^2 \\
&-2\csc\theta\, g_{ij}\left(\sin\theta\,\partial_\theta\phi^i - f g^{il}U_A\partial_l X^A\right)\left(\sin\theta\,\partial_\theta\phi^j - f g^{jk}U_B\partial_k X^B\right) \\
&+2\csc\theta\, f^{-2}G_{AB}\left(\sin\theta\,\partial_\theta\chi^A + f^2G^{AC}U_C\right)\left(\sin\theta\,\partial_\theta\chi^B + f^2G^{BD}U_D\right) \\
&+2\partial_\theta\left(\sin\theta f^{-1}\partial_\theta f - 2U_A\,\chi^A - 2f\,U_A X^A\right) \\
&\qquad +4\chi^A\partial_\theta U_A + 4fX^A\partial_\theta U_A\Bigg]\ .
\end{aligned} \tag{3.11}
$$

When requiring stationarity of S_1 with respect to variations of the fields, the last two terms in (3.10) and (3.11) vanish. Thus, up to a total derivative term, S_1^a is expressed in terms of squares of first-order flow equations which result in

$$
\begin{aligned}
\partial_\tau f^{-1} &= \frac{2}{3}V_A X^A\ , \\
\partial_\tau\chi^A &= f^2 G^{AC}V_C\ , \\
\partial_\tau\phi^i &= -f g^{il}V_A\partial_l X^A\ .
\end{aligned} \tag{3.12}
$$

In the same result from S_1^b, we can obtain:

$$\begin{aligned}
\sin\theta\, f^{-2}\partial_\theta f &= \frac{2}{3}U_A X^A\,,\\
\sin\theta\,\partial_\theta \chi^A &= -f^2 G^{AC}U_C\,,\\
\sin\theta\,\partial_\theta\phi^i &= f g^{il}U_A\partial_l X^A\,.
\end{aligned}\tag{3.13}$$

Eqs. (3.12) and (3.13) describe the evolution of the spacetime metric and the moduli fields in the background of five dimensional Gibbons-Hawking base space, which are analogous to the flow equations for supersymmetric black holes in asymptotically flat spacetime in five dimensions derived in [11, 12]. It is worth pointing out that these first-order equations were derived without using supersymmetry, therefore, attractor flow equations (3.12) and (3.13) can including the supersymmetric and non-supersymmetric case. In next section, we will rewrite above two set of equations into one compact form.

Also, we rewrite S_2 as a sum of squares, as follows.

$$\begin{aligned}
S_2 = -\int & dt\,dr\,d\theta\,d\phi\,d\psi\, r^{-1} f\sin\theta\, G_{AB}\\
&\Big[\Big(r\,\partial_r\eta_5^A - s\,(\eta_5^A - \csc\theta\,\partial_\theta\eta_4^A)(r\,\partial_r\eta_5^B - s\,(\eta_5^B - \csc\theta\,\partial_\theta\eta_4^B)\\
&\quad +(\partial_\theta\eta_5^A - s\,r\csc\theta\,\partial_r\eta_4^A)(\partial_\theta\eta_5^B - s\,r\csc\theta\,\partial_r\eta_4^B\Big)\Big]\,.
\end{aligned}\tag{3.14}$$

thus, we obtain one additional first-order equations about η^I following the stationarity of S_2:

$$r\,\partial_r\eta_5^A = s\,(\eta_5^A - \csc\theta\,\partial_\theta\eta_4^A)\,,\tag{3.15a}$$

$$\partial_\theta\eta_5^A = s\,r\csc\theta\,\partial_r\eta_4^A\,.\tag{3.15b}$$

Also, we rewrite the eqs. (3.15) in compact form:

$$\nabla\times\hat{\eta}_4^I = -\nabla(H\eta_5^I)\,,\tag{3.16}$$

with $H = r^{-1}$. We reproduces the eqs. (2.15) precisely.

Next, we rewrite S_3 as a sum of squares, as follows. Using the definition in [11]

$$\chi^A = -sf\,X^A\,,\tag{3.17}$$

with $s = 1$, Then, we obtain for S_3,

$$\begin{aligned}
S_3 = -\int & dt\,dr\,d\theta\,d\phi\,d\psi\, r^{-1} f^3\sin\theta\\
&\Big[\Big(r\,\partial_r w_5 + s(w_5 - \csc\theta\,\partial_\theta w_4) + 3\,s\,r f^{-1}X_A\partial_r\eta_5^A\Big)\Big(r\,\partial_r w_5\\
&\qquad + s(w_5 - \csc\theta\,\partial_\theta w_4) + 3\,f^{-1}X_A(\eta_5^A - \csc\theta\partial_\theta\eta_4^A)\Big)\\
&\qquad +(\partial_\theta w_5 + s\,r\csc\theta\,\partial_r w_4 + 3\,s\,f^{-1}X_A\partial_\theta\eta_5^A)(\partial_\theta w_5 + s\,r\csc\theta\,\partial_r w_4\\
&\qquad\qquad +3\,r\csc\theta f^{-1}X_A\partial_r\eta_4^A)\Big]\,.
\end{aligned}\tag{3.18}$$

Then, another additional first-order flow equations following from the stationarity of S_3 are

$$r\,\partial_r w_5 + s(w_5 - \csc\theta\,\partial_\theta w_4) = -3\,s\,r f^{-1} X_A \partial_r \eta_5^A\ , \tag{3.19a}$$
$$r\,\partial_r w_5 + s(w_5 - \csc\theta\,\partial_\theta w_4) = -3\,f^{-1} X_A (\eta_5^A - \csc\theta \partial_\theta \eta_4^A)\ , \tag{3.19b}$$
$$\partial_\theta w_5 + s\,r\csc\theta\,\partial_r w_4 = -3\,s\,f^{-1} X_A \partial_\theta \eta_5^A\ , \tag{3.19c}$$
$$\partial_\theta w_5 + s\,r\csc\theta\,\partial_r w_4 = -3\,r\csc\theta f^{-1} X_A \partial_r \eta_4^A\ . \tag{3.19d}$$

Considering eqs. (3.15), we can obtain:

$$\nabla\times\hat{\omega}_4 = H\nabla\omega_5 - \omega_5\nabla H + 3\,H\big(f^{-1}X_A\big)\nabla\eta_5^A\ , \tag{3.20}$$

with $H = r^{-1}$. It is obvious to see that the gauge fields A^I and one-form ω are subject to the constraint of equations (3.16) and (3.20).

So far we have discussed the bosonic part of the supergravity action in five dimensional Gibbons-Hawking base. Assuming the stationary of action, we obtain the general first-order flow equations (3.12) and (3.13), which describe the evolution of the metric and the moduli fields in the background of general five dimensional solution. Meanwhile, the constraint (3.16) and (3.20) can be obtain. Since the present discussion is that it does not rely on supersymmetry, the above conclusion can include both BPS and non-BPS solutions.

4. Some properties of attractor flow equations for supersymmetric black rings

In order to solve the flow equations (3.12) and (3.13), we need rewrite two equations into one compact form. First, we introduce a anszta:

$$f^{-1}X_I = \zeta_I\ , \tag{4.1}$$

where $\zeta_I = \zeta_I(r,\theta)$. We take the gradient of this equation with respect to the base space $\mathbb{E}^3$ and obtain:

$$f^{-1}\nabla X_I + X_I \nabla f^{-1} = \nabla\zeta_I\ . \tag{4.2}$$

Using the relation $X_I X^I = 1$, we also have: $X^I\nabla X_I = \nabla X^I X_I = 0$, and consequently, we write eqs. (4.2) as:

$$\nabla f^{-1} = X^I \nabla\zeta_I\ . \tag{4.3}$$

Comparing eqs. (4.2) with eqs. (3.12) and (3.13), we obtain

$$V_I = -\frac{3}{2} r^2\,\partial_r\,\zeta_I\ , \qquad U_I = -\frac{3}{2}\sin\theta\,\partial_\theta\,\zeta_I\ . \tag{4.4}$$

So, we combine eqs. (3.12) and (3.13) into

$$\nabla f^{-1} = X^I \nabla\zeta_I\ , \tag{4.5a}$$
$$\nabla\chi^A = \frac{3}{2} f^2 G^{AI} \nabla\zeta_I\ , \tag{4.5b}$$
$$\nabla\phi^i = -\frac{3}{2} f g^{il} \nabla\zeta_I\,\partial_l X^I\ . \tag{4.5c}$$

These equations which are called the first-order attractor flow equations are one of main results in this paper. The flow equations describe the evolution of the spacetime metric and the moduli. Remarkably, we can observe that equations (4.5) take the form analogous to that of a gradient flow of black holes in five dimensions derived in [11, 12] .

So far we obtain the first-order flow equations in addition the constraint eqs. (3.16) and (3.20) for the general solutions following the stationarity of actions S. We note that in the above consideration we have not considered the supersymmetric properties of the solutions, therefore the solution would be BPS or non-BPS. When including the supersymmetry, we get $f^{-1}X_I = \zeta_I = \frac{1}{24}H^{-1}C_{IPQ}K^PK^Q + L_I$ by solving the BPS equations (2.11). So, the flow equations (4.5) are solved

$$f^{-1} = \left(\frac{1}{24}H^{-1}C_{IPQ}K^PK^Q + L_I\right)X^I \; , \tag{4.6a}$$

$$\chi^A = -s\, f X^A \; , \tag{4.6b}$$

$$f^{-1}X_I = \frac{1}{24}H^{-1}C_{IPQ}K^PK^Q + L_I \; , \tag{4.6c}$$

where $H^{-1} = r$, and L_I, K^I are harmonic functions on $\mathbb{E}^3$ and are given by eqs. (2.21). The same solutions of supersymmetric (multi)black rings have been obtain in [7, 14, 15].

We would like to emphasize that when considering the supersymmetric configuration (2.11), the flow equations (4.5) have the following properties:

(i) Integrating eq. (4.5a) on base space $\mathbb{E}^3$, we can introduce a term $Z_e(V)$ as

$$Z_e(V) = \frac{3}{4\pi^2}\int_{\partial V} d\overrightarrow{\mathbf{S}} \cdot \nabla f^{-1} \; , \tag{4.7}$$

where ∂V is a closed hypersurface in base space $\mathbb{E}^3$. Using the outward pointing unit normal vector $\mathbf{n}$, we get

$$\begin{aligned} Z_e(V) &= \frac{3}{4\pi^2}\int_{\partial V} dS\, f^{-2}n^m\partial_m f \; , \\ &= \frac{1}{2\pi^2}\int_{\partial V} dS\, f^{-1}X^In^mE_{mI} \; , \end{aligned} \tag{4.8}$$

where

$$E_{mI} \equiv G_{IJ}F^J_{m\hat{t}}, \;\; F^I_{m\hat{t}} = f^{-1}\partial_m(fX^I) \; . \tag{4.9}$$

It is obvious to show that the definition (4.7) agrees precisely with the electronic central charge in [10].[1] We can consider that Z_e is the electric charge corresponding to the graviphoton.

(ii) There is another form of the attractor formula that is cast entirely in terms of the moduli space. To derive it, we multiply the term $\partial_i X^I$ on both sides of (4.5c), we can write the result as

$$\nabla X^I = -\frac{3}{2}fg^{il}\nabla\zeta_J\,\partial_l X^J\,\partial_i X^I. \tag{4.10}$$

[1]Note that we choose the three-dimensional submanifold of four-dimensional hyper-Kähler manifold $\mathcal{M}$ in this section, which is the only difference comparing with the choice in [10].

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Using the relation (2.6), we get

$$\nabla X^I = -\frac{3}{2} f G^{IJ} \nabla\zeta_K D_J X^K \,, \tag{4.11}$$

where the covariant derivative is defined as $D_I = \partial_I - \frac{1}{3}X_I$.

(iii) In condition of supersymmetric solutions, we take the divergence of eq. (4.5a) and obtain:

$$\begin{aligned} \nabla\cdot\nabla f^{-1} &= \nabla\cdot(X^I\nabla\zeta_I) \\ &= \nabla X^I\cdot\nabla\zeta_I + X^I\nabla\cdot\nabla\zeta_I \\ &= -\frac{2}{3}f^{-1}G_{IJ}\nabla X^I\cdot\nabla X^J + \frac{1}{6}C_{IJK}X^I\Theta^J\cdot\Theta^K \,, \end{aligned} \tag{4.12}$$

where we used (2.6) and second BPS equations (2.11) to arrive at the third line. We can rewrite (4.12) as

$$\nabla^m(f^{-1}X^I E_{mI}) = f^{-1}G_{IJ}\nabla X^I\cdot\nabla X^J - \frac{1}{4}C_{IJK}X^I\Theta^J\cdot\Theta^K \,, \tag{4.13}$$

where E_{mI} take the definition (4.9). This equation agrees precisely with flow equations obtain by Kraus and Larsen in [10].

In the rest of this section we give a special case as an example. We take the compactification manifold to be T^6, In this case $C_{IJK} = 1$ if (IJK) is a permutation of (123), and $C_{IJK} = 0$ otherwise. The metric G_{IJ} is

$$G_{IJ} = \frac{1}{2}\mathrm{diag}((X^1)^{-2},(X^2)^{-2},(X^3)^{-2}) \,. \tag{4.14}$$

When M=1 and $x_1 = 0$ in (2.21), i.e. all of the harmonic functions in $\mathbb{E}^3$ are centred at the origin. With the metric (2.20) , we find $\chi^I = H_I^{-1} = \lambda_I + Q_I/r$, and the gauge potential

$$A^I = H_I^{-1}(dt+\omega), \;\; \omega_5 = -\frac{J}{2r}, \;\; \omega_4 = 0 \,, \tag{4.15}$$

where $J = \frac{1}{2}(q_1Q_1 + q_2Q_2 + q_3Q_3 - q_1q_2q_3)$. This is the BMPV black hole with three independent charges Q_i and angular momenta $J_\phi = J_\psi = J$ [20]. Inserting (4.15) into the flow equations (4.5), we can obtain

$$\partial_\tau f^{-1} = Z_e \,, \tag{4.16a}$$

$$\partial_\tau\phi^i = -\frac{3}{2}fg^{il}\partial_l Z_e \,, \tag{4.16b}$$

with $Z_e = X^I Q_I$. We can use an equivalent way to find the attractor point of these equations, which is that one can find the fixed values of the moduli by extremizing the central charge Z_e. Extremizing the central charge with respect to the fixed moduli means that we impose $\partial_i Z_e = 0$. We shall assume that the charges Q_I are chosen such that at the attractor point, $Z_e = Z_e^* \neq 0$. Thus, equation (4.16a) may be easily integrated near the horizon,

$$f^{-1} \sim Z_e^*/r \, . \tag{4.17}$$

The Bekenstein-Hawking entropy is one quarter of the horizon area,

$$S_{BH} = 2\pi \lim_{r\to 0} \sqrt{(r\,f^{-1})^3 - J^2} = 2\pi \sqrt{(Z_e^*)^3 - J^2} \, . \tag{4.18}$$

We reproduce the BMPV black hole entropy in five dimensions which have been obtained in [20].

5. Conclusion

In this paper, by the use of the stationarity of actions , we have obtained the first-order attractor flow equations for the general solutions of motion equations for $N = 2$ supergravity in five dimensional Gibbons-Hawking space. Meanwhile, we also get the constraint (3.16) and (3.20) which determine the gauge field A^I and one-form ω. Furthermore, when considering the supersymmetry we obtain the first-order flow equations for supersymmetric (multi)black rings. It is also showed that the supersymmetric solution with Gibbons-Hawking base is specified by $2n+2$ harmonic functions H, K^I, L_I and B on the flat space $\mathbb{E}^3$. Using the very special geometry, We analyze the equation (4.5a) in first-order flow equations and find that the integrate of r.h.s. of (4.5a) agrees precisely with the electronic central charge Z_e in [10]. Moreover, taking the divergence of (4.5c), we can reproduce the second-order flow equation which have been obtained by Kraus and Larsen in [10]. A particular case, BMPV black hole which is the limits of the supersymmetric black ring solution, is presented in the last.

Acknowledgments

We would like to thank C. Cao, Q. J. Cao, Y. J. Du, J. L. Li, Q. Ma and Y. Xiao for useful discussions. The work is supported in part by the NNSF of China Grant No. 90503009, No. 10775116, and 973 Program Grant No. 2005CB724508.

A. Evaluating the action in five dimensions

The square root of the determinant of the metric (3.1), (2.20) is

$$\sqrt{-G} = \frac{r\sin\theta}{f} \, . \tag{A.1}$$

The inverse metric reads

$$G^{MN} = \begin{pmatrix} -1/f^2 + f\left(\frac{w_5^2}{r} + \frac{w_4^2}{r\sin^2\theta}\right) & 0 & 0 & -\frac{fw_4}{r}\frac{1}{\sin^2\theta} & f\left(-\frac{w_5}{r} + \frac{w_4}{r}\frac{\cos\theta}{\sin^2\theta}\right) \\ 0 & rf & 0 & 0 & 0 \\ 0 & 0 & f/r & 0 & 0 \\ -\frac{fw_4}{r}\frac{1}{\sin^2\theta} & 0 & 0 & \frac{f}{r}\frac{1}{\sin^2\theta} & -\frac{f}{r}\frac{\cos\theta}{\sin^2\theta} \\ f\left(-\frac{w_5}{r} + \frac{w_4}{r}\frac{\cos\theta}{\sin^2\theta}\right) & 0 & 0 & -\frac{f}{r}\frac{\cos\theta}{\sin^2\theta} & f\left(\frac{1}{r} + \frac{1}{r}\frac{\cos^2\theta}{\sin^2\theta}\right) \end{pmatrix} \tag{A.2}$$

and the Ricci scalar is

$$
\begin{aligned}
R = \frac{1}{2r^2 f}\Big[& f^5 (w_5^2 + \partial_\theta^2 w_5 - 2\csc\theta\, w_5 \partial_\theta w_4 + \csc^2\theta\, \partial_\theta w_4^2 \\
& + r^2\, \partial_r w_5^2 + r^2 \csc^2\theta\, \partial_r w_4^2) - 5r(\partial_\theta^2 f + r^2 \partial_r^2 f) \\
& + 2rf(\cot\theta\, \partial_\theta f + \partial_{\theta\theta} f + 2r\partial_r f + r^2 \partial_{rr} f)\Big] .
\end{aligned} \tag{A.3}
$$

Inserting the ansatz

$$
A^I = \chi^I(r,\theta)\,(dt + w) + \eta^I , \tag{A.4}
$$

into the gauge kinetic term in (2.1) yields

$$
\begin{aligned}
-\frac{1}{2}\sqrt{-G}\, G_{AB} F^A_{MN} F^{BMN} = -\frac{\sin\theta}{2rf^2} G_{AB}\Big[& 2f^3 \Big(\eta_5^A \eta_5^B + \partial_\theta \eta_5^A \partial_\theta \eta_5^B - 2\csc\theta\, \eta_5^A \partial_\theta \eta_4^B \\
& + \csc^2\theta \partial_\theta \eta_4^A \partial_\theta \eta_4^B + r^2 \partial_r \eta_5^A \partial_r \eta_5^B + r^2 \csc^2\theta \partial_r \eta_4^A \partial_r \eta_4^B \\
& + 2\chi^A (\partial_\theta \eta_5^B\, \partial_\theta w_5 - \csc\theta \eta_5^B\, \partial_\theta w_4 + \csc^2\theta\, \partial_\theta \eta_4^B\, \partial_\theta w_4 \\
& - \csc\theta w_5\, \partial_\theta \eta_4^B + \eta_5^B w_5 + r^2\, \partial_r \eta_5^B \partial_r w_5 \\
& + r^2 \csc^2\theta\, \partial_r \eta_4^B \partial_r w_4) + \chi^A \chi^B (w_5^2 + \partial_\theta w_5^2 \\
& - 2\csc\theta w_5 \partial_\theta w_4 + \csc^2\theta \partial_\theta w_4^2 + r^2 \partial_r w_5^2 + r^2 \csc^2\theta \partial_r w_4^2)\Big) \\
& - 2r \left(\partial_\theta \chi^A \partial_\theta \chi^B + r^2 \partial_r \chi^A \partial_r \chi^B \right)\Big] .
\end{aligned} \tag{A.5}
$$

The Chern-Simons term in (2.1) evaluates to

$$
\begin{aligned}
-\frac{1}{6V} C_{ABC}\, F^A \wedge F^B \wedge A^C = \frac{1}{3V} C_{ABC}\Big[& \chi^A \chi^B \chi^C \Big((w_5 \sin\theta - \partial_\theta w_4) \partial_r w_5 + \partial_r w_4 \partial_\theta w_5 \Big) \\
& - \frac{3}{2}\chi^A \chi^B \Big(\partial_r w_5 \partial_\theta \eta_4^C - \eta_5^C \sin\theta \partial_r w_5 - \partial_\theta w_5 \partial_r \eta_4^C \\
& + \partial_\theta w_4 \partial_r \eta_5^C - \partial_r w_4 \partial_\theta \eta_5^C - w_5 \sin\theta \partial_r \eta_5^C \Big) \\
& + 3\chi^A \Big(\partial_r (\eta_5^B \cos\theta + \eta_4^B) \partial_\theta \eta_5^C - \partial_\theta (\eta_5^B \cos\theta + \eta_4^B) \partial_r \eta_5^C \Big) \\
& + \frac{1}{2}\partial_r \Big(\chi^A \chi^B \eta_5^C \partial_\theta (w_5 \cos\theta + w_4) - \chi^A \chi^B (\eta_5^C \cos\theta \\
& + \eta_4^C) \partial_\theta w_5 + 2\chi^A (\eta_5^B \partial_\theta \eta_4^C - \eta_5^B \eta_5^C \sin\theta - \partial_\theta \eta_5^B \eta_4^C) \Big) \\
& - \frac{1}{2}\partial_\theta \Big(\chi^A \chi^B \eta_5^C \partial_r (w_5 \cos\theta + w_4) - \chi^A \chi^B (\eta_5^C \cos\theta \\
& + \eta_4^C) \partial_r w_5 + 2\chi^A (\eta_5^B \partial_r \eta_4^C - \partial_r \eta_5^B \eta_4^C) \Big) \Big] \\
& \times dt \wedge dr \wedge d\theta \wedge d\phi \wedge d\psi .
\end{aligned} \tag{A.6}
$$

When defining

$$
\chi^A = -sf\, X^A , \tag{A.7}
$$

the terms in (A.6)

$$\frac{1}{3V}C_{ABC}\Big[+3\chi^A\Big(\partial_r(\eta_5^B\cos\theta+\eta_4^B)\partial_\theta\eta_5^C-\partial_\theta(\eta_5^B\cos\theta+\eta_4^B)\partial_r\eta_5^C\Big)\Big]$$
$$=\left(2G_{AB}sf-9\frac{X_AX_B}{V^2}sf\right)\Big(\partial_r(\eta_5^B\cos\theta+\eta_4^B)\partial_\theta\eta_5^A-\partial_\theta(\eta_5^B\cos\theta+\eta_4^B)\partial_r\eta_5^A\Big). \tag{A.8}$$

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